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At What Energy Does Gravity Unite with Grand Unified Theories in the Early Universe?

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Unification of gravity with GUTs is usually expected at the Planck energy $E_{\mathrm{Pl}} \approx 10^{19}$ GeV. However, the vastly different values of the two couplings at E_{Pl} ($\alpha_{\mathrm{GUT}} \ll 1$, $\alpha_{\mathrm{grav}} \approx 1$) would make such unity (at E_{Pl}) implausible unless there is a drastic change in the behavior of either gravity or GUTs around E_{Pl}. We picture gravity and GUTs to be unified at energies $> E_{\mathrm{Pl}}$ with a single dimensionless coupling constant ($\alpha_U \ll 1$) and described by a scale-invariant action quadratic in the Weyl curvature (with Yang-Mills fields). Breaking of scale invariance at E_{Pl} then separates the interactions into gravity, now described by a Hilbert action with a dimensional G and GUTs woth a dimensionless α_{GUT} and YM action. Problems with Klein-Kaluza unification of gravity with GUTs are also discussed in this context.

1. INTRODUCTION

Most of the grand unified theories (GUTs) involve the unification of weak, electromagnetic, and strong interactions at energies around 10^{-3} to 10^{-4} times E_{Pl}, where $E_{\mathrm{Pl}} = (\hbar c^5/G)^{1/2} \approx 10^{19}$ GeV is the Planck energy. Thus, the three independent coupling constants characterizing these interactions at low energies become equal at an energy $\sim 10^{15}$ GeV (Langacker, 1981), the interactions being described by a scale-invariant action quadratic in the Yang-Mills field strength. Above this unification energy (i.e., $\sim 10^{-4} E_{\mathrm{Pl}} \approx 10^{15}$ GeV) it is usually stated that these three interactions are characterized by a *single dimensionless* coupling constant α_{GUT}, which is variously estimated in a model-dependent way (Llewellen Smith, 1983) as $\alpha_{\mathrm{GUT}} \approx 1/40$ at the unification energy, but in any case it is $\ll 1$. Moreover, α_{GUTs} is energy dependent and is expected to continue to *decrease* logarithmically with increasing energies (i.e., in accordance with asymptotic freedom typical of non-abelian gauge theories with the coupling constant

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tending to zero at very large energies E). Thus, at Planck energies, one would expect that α_{GUT} would be less than 1/40 and closer to 1/100. However, in the meantime, in totally contrasting behavior, the gravitational coupling constant ($\alpha_{\mathrm{grav}} = GE^2/\hbar c^5$) continues to rise with increasing energy as E^2. At the GUT unification energy of $\sim 10^{15}$ GeV, α_{grav} is only $\sim 10^{-8}$, as compared to $\alpha_{\mathrm{GUT}} \approx 1/40$. But at the Planck energy it rises to $\alpha_{\mathrm{grav}} \approx 1$, i.e., it becomes much higher now than $\alpha_{\mathrm{GUT}} \approx 10^{-2}$, which continues to decrease.

It is also usually supposed that at the Planck energy (where $\alpha_{\mathrm{grav}} \approx 1$), gravity gets unified with the other three interaction, i.e., with GUTs (Ellis, 1983). However, if they are indeed unified at E_{Pl}, one would expect both gravity and GUTs to be characterized by a single dimensionless coupling constant at E_{Pl}. But as noted above, if the gravity and GUT coupling constants continue to behave in an opposite manner with respect to rising energy between the GUT unification energy and E_{Pl}, they would have vastly different values at the Planck energy.

So how does one achieve unification of gravity and GUTs at E_{Pl} in such a paradoxical situation, unless there is some drastic discontinuity in either the GUT or gravity interaction at or around $E = E_{\mathrm{Pl}}$?

The GUT interaction (incorporating the strong and electroweak forces) is described at high energies by the YM field with *scale-invariant* actions *quadratic* in the fields with *dimensionless* coupling constants tending to zero at highest energies. Analogously, one would expect gravity at very high energies (like those occurring in the early universe) to be also described by a scale-invariant gauge theory with an action quadratic in the curvature and with a dimensionless coupling constant. The Hilbert action $R/16\pi G$ is linear in the curvature, has a dimensional constant G, and is not scale-invariant [i.e., under the transformation $g'_{\mu\nu} \to \lambda^2 g_{\mu\nu}$, where λ is a function of the position $\lambda = \lambda(x)$], but is only invariant under the group of general coordinate transformations (GCT). It describes gravity well at low energies (i.e., large distances), but its dimensional coupling constant causes its bad behavior at high energies (the cross sections for quantum gravitational processes becoming singular at large E). This is analogous to weak interactions at low energies being described by the dimensional Fermi constant G_{F}, which again causes singular behavior at large energies. But the Fermi interaction is only an effective low-energy interaction, the weak interaction at high energies ($>10^2$ GeV) being characterized by a dimensionless coupling constant, identical with the electromagnetic constant above these energies, resulting in electroweak unification. A similar situation can be expected for gravity, the Hilbert action with a dimensional G being the effective low-energy counterpart of a unified theory of gravity and other interactions above E_{Pl}.

2. UNIFICATION OF GRAVITY WITH GUTS

We suppose that the unification occurs not at E_{Pl}, but at energies higher than E_{Pl} when both gravity and GUTs are described by a *single dimensionless* coupling constant α_U (which is $\ll 1$) and a quadratic action that is both scale- and GCT-invariant. Then at $E = E_{Pl}$ (i.e., as the universe expands), the scale invariance is broken (Sivaram, 1986a), inducing the Hilbert term (linear in R) and an induced dimensional gravitational constant, which is strong (~ 1) at E_{Pl}; the interactions separate out into gravity and GUTs characterized by differently behaved (with respect to energy) coupling constants at energies $E < E_{Pl}$.

An appropriate action for the unified description of gravity and GUTs at energies above E_{Pl} with a single dimensionless α_U is given by

$$\int \alpha_U \, d^4x \, (-g)^{1/2} (W^2 - \tfrac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}) \tag{1}$$

where W is the Weyl curvature scalar, related to the Riemann scalar R by

$$W = R - 6(A^\mu{}_{,\mu} - A^\mu A_\mu)$$

and A is the Weyl four-vector, gauge transforming as $A^a_\mu \to A^a_\mu + \lambda_{,\mu}$, where λ is the scale parameter. The index a in the Yang–Mills field strength F can take values depending on the groups and multiplets considered. The above action is both scale- and GCT-invariant. We note that only an action quadratic in the curvature (field strength) would have a dimensionless coupling constant, ruling out use of other powers of the curvature (Sivaram, 1986a,b). For the action (1), we have the variational principle:

$$0 = \alpha_U \int [W^2 \delta(-g)^{1/2} + 2 W \delta W (-g)^{1/2} + \delta(F^a_{\mu\nu} F^{a\mu\nu} (-g)^{1/2})] \, d^4x \tag{2}$$

varying $g_{\mu\nu}$, A_μ independently. To *break* the scale invariance, we incorporate a characteristic scale (energy or length) at $E = E_{Pl}$, where we can set

$$W = \Lambda_{Pl}, \qquad \Lambda_{Pl} \approx E^2_{Pl}$$

Then the variational principle given by (2) becomes

$$\delta \int \left(W + \frac{1}{2\Lambda_{Pl}} F^a_{\mu\nu} F^{a\mu\nu} - \tfrac{1}{2}\Lambda_{Pl} \right) (-g)^{1/2} \, d^4x = 0 \tag{3}$$

Now substituting the relation connecting W and R as given earlier and using the transformations

$$A'_\mu = (\Lambda_{Pl})^{-1/2} A_\mu, \qquad F'_{\mu\nu} = (\Lambda_{Pl})^{-1/2} F_{\mu\nu}$$

we have

$$\delta \int (-g)^{1/2} \, d^4x \, [KR + \alpha_{GUT} F'^a_{\mu\nu} F'^{a\mu\nu} - \Lambda_{Pl}(\tfrac{1}{2} + 6 A'_\mu A'_\mu)] = 0 \tag{4}$$

Here K is a dimensional constant, which is related to Λ_{Pl} ($K \propto \Lambda_{\mathrm{Pl}}^{-1}$) and α_{GUT} and is now introduced into the action as a multiplier of the Hilbert term brought into the action by the breaking of the scale invariance at $E = E_{\mathrm{Pl}}$, with of course an induced cosmological constant term $\sim\Lambda_{\mathrm{Pl}}$ and mass terms for the gauge fields. Thus, below Planck energies, gravity and GUTs separate out, the GUT interaction being now characterized by a dimensionless α_{GUT} coupling constant and the Yang–Mills part of the action, which is still scale-invariant.

The gravitational interaction is now described by the Hilbert term with a dimensional coupling K that is large at the Planck energy, i.e., as $GE_{\mathrm{Pl}}^2/\hbar c \sim 1$. This is analogous to the strong interaction being described by a coupling constant ~ 1 (i.e., becoming strong) at energies around the QCD scale-breaking parameter Λ_{QCD} (with an effective chiral-invariant action describing the strong interaction at low energies) when the pion bound states form from fundamental quark fields (i.e., the interaction becoming strong with coupling ~ 1 at Λ_{QCD} leads to quark confinement). At high energies, the strong interactions are described by scale-invariant QCD action with coupling $\ll 1$ decreasing with rising energy (Politzer, 1973). Analogously, we note that α_U, the single dimensionless constant for both GUTs and gravity at energies above E_{Pl}, is much less than one and decreases with energy.

After the scale breaking at E_{Pl}, α_{GUT} continues to be $\ll 1$ and changes slowly with energy, while gravity below E_{Pl} has a dimensional constant $G \sim (16\pi K)^{-1}$. At $E_{\mathrm{GUT}} \approx 10^{-4} E_{\mathrm{Pl}}$ the GUT interaction further splits into strong and electroweak interactions with different dimensionless couplings, while the gravitational interaction coupling now decreases with decreasing energy as E^2.

The large induced cosmological term Λ_{Pl} in equations (3) and (4) can cause an exponential expansion epoch for the early universe (Sivaram, 1986a), i.e., create a deSitter inflationary phase with negative pressure, just after the Planck epoch. It may be noted that the ground state of many supergravity models that have GUT unification just below Planck energies is an anti-deSitter space characterized by a large *negative* cosmological constant of the same magnitude, i.e., $-\Lambda_{\mathrm{Pl}}$. This could cancel out the induced Λ_{Pl} once the exponential expansion sets in and supersymmetry multiplets manifest out of the vacuum.

3. PROBLEMS WITH KLEIN–KALUZA UNIFICATION OF GRAVITY WITH GUTS

Another paradoxical situation regarding the coupling constants of gravity and GUTs arises in the Klein–Kaluza (KK) type of unification, where the action for pure gravity in $D>4$ space-time dimensions,

$$1/16\pi G_D \int d^Dx\,(g^{(D)})^{1/2}(R-2\Lambda)$$

becomes, after the process of dimensional reduction, the action for gravity coupled to a gauge theory in *four* dimensions,

$$1/16\pi G_4 \int d^4x\,(g^{(4)})^{1/2}(R^{(4)}-2\Lambda)-\tfrac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}$$

In particular, in five-dimensional KK, we can identify the fifth component of the canonical momentum P^5 conjugated to the fifth coordinate x^5 with electric charge Q as $P^5=c(16\pi G)^{1//2}Q$. Alternatively, x^5 and Q can be regarded as two dynamical operators satisfying the quantum commutation $[x^5, Q]=i\hbar$, which gives the extension of the R^5 coordinate for a fundamental electric charge e as

$$R^5\sim \hbar\sqrt{G}/ec \tag{5}$$

Similarly, to relate electroweak coupling constants to the geometry of some internal space, we can choose 3-dimensional Taub space as the internal space with metric $ds^2=-dt^2+\sum_{a=1}^{3} r_a^2(S^a)^2$, two of the principal radii of curvature r_a being equal ($r_1=r_2\neq r_3$). The S^a are basis 1-forms on the 3-sphere. The isometry group of Taub space is $SU(2)\times U(1)$, i.e., the electroweak gauge group. If g, g' are the coupling constants of the gauge fields of $SU(2)$ and $U(1)$, we can obtain the relations

$$r_1=\frac{\hbar\sqrt{G}}{cg}\left[1+\frac{1}{3}\left(\frac{r_3^2}{r_1^2}-1\right)^{1/2}\right]^{1/2},$$
$$\frac{r_3^2}{r_1^2}=\frac{\hbar^2}{c^2}\frac{G}{g'^2},\qquad \frac{g'}{g}=\tan\theta_w \tag{6}$$

Thus, the non-Abelian gauge coupling constants are *inversely* proportional to the "average size" of the internal space. Generally, the radius of compactification R_c of internal space is

$$R_c\approx(\hbar/c)\sqrt{G}/g\approx\alpha_g^{-1/2}R_{\rm Pl} \tag{7}$$

where $R_{\rm Pl}=(\hbar G/c^3)^{1/2}$ is the Planck length. Thus, for the KK unification of gravity with GUTs, the size of the compactified internal space is

$$R_c\approx\alpha_{\rm GUT}^{-1/2}R_{\rm Pl}\qquad\text{or}\qquad R_{\rm Pl}/R_c\approx\alpha_{\rm GUT}^{1/2} \tag{8}$$

We have $\alpha_{\rm GUT}\ll 1$. For example, for the $SO(14)$ gauge group, $\alpha_{\rm GUT}=1/16$ and for supersymmetric GUTs, $\alpha_{\rm GUT}=1/24$. Thus, the unification does not occur at $R_{\rm Pl}$ (where $\alpha_g=1$) (corresponding to $E_{\rm Pl}$), but at R_c, corresponding to an energy $E_c\sim\hbar c/R_c$ at which $\alpha_{\rm GUT}\approx\alpha_{\rm grav}$. For SUSY GUTs, the energy at which the coupling constants of gravity and of GUTs are equal is $\sim 2\times 10^{18}$ GeV, corresponding to $E_c \approx 0.2 E_{\rm Pl}$. Now, in cosmological models with extra dimensions (i.e., $4+d$), while the four space-time dimensions are expanding, the d extra space dimensions [$d=7$ to accommodate $SU(3)\times SU(2)\times U(1)$] are contracting (i.e., getting compactified) (Appelquist and Chodas, 1983).

Now, contraction of internal space would suggest from equations (6)–(8) that the coupling constant $\alpha_{\rm GUT}$ should *increase* with decreasing R_c or *increasing* energy E_c. This would be contrary to asymptotic freedom in unified gauge theories, where $\alpha_{\rm GUT}$ *decreases* with increasing energy. One way to understand this contradiction is to suppose that after the unification takes place at $E_c \approx 0.2 E_{\rm Pl}$ the single coupling constant describing both gravity and GUTs now increases with energy (i.e., with decreasing radius of compactification) rising up to ~ 1 at $E_{\rm Pl}$.

It must be observed in this context that the unified coupling constant in KK compactification is a dimensional gravitational constant G_D in D dimensions, which would consequently make the coupling rise with energy ($\sim E^{D-2}$). At the classical level, the D-dimensional and 4-dimensional gravitational constants are related by $G_4 = G_D/R_c^N\Omega_N$, $N=D-4$, where Ω^N is the volume of the compactified unit N-sphere. One finds for $SO(14)$ that $R_c/G_4^{1/2} \approx = 21$. But the identification of G_4 with the Planck length is only valid classically, and quantum corrections would make G_4 *different* from G.

Again, if there are very many heavy particles with masses between $M_{\rm GUT}$ and $M_{\rm Pl}$, their effect on the GUT renormalization group equations can be so large as to *reverse* the normal asyumptotic freedom trend for $\alpha_{\rm GUT}$ to decrease with increasing energy and instaead to make $\alpha_{\rm GUT}$ *increase* as the energy scale increases, so that $\alpha_{\rm GUT} \approx \alpha_{\rm grav} \approx 1$ at $E_{\rm Pl}$ where unification occurs. A particular model where a very large number of scalar fields is required for consistent KK unification is available (Candelas and Weinberg, 1984). Here the KK Lagrangian is coupled to n massless scalar fields in $D=N+4$ dimensions. It turns out that the gauge coupling g is then related to N, R_c, and n, and to get a reasonable value for g ($\alpha_{\rm GUT} \approx g^2/\hbar c$) it turns out that n is very large, i.e., $>10^4$. The presence of so many scalar particles may reverse the trend for $\alpha_{\rm GUT}$ to decrease with rising energy, so that beyond E_C, both gravity and GUTs are described by a single coupling constant rising with energy.

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